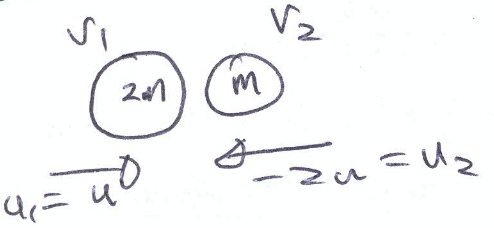
1996 5(a):



PCM: $2m(u) + m(-2u) = 2mv_1 + mv_2$

$$\boxed{0 = 2v_1 + v_2} \quad (1) \quad (5)$$

NLR $\quad v_2 - v_1 = -e(u_2 - u_1)$

$\Rightarrow \quad v_2 - v_1 = -e(-2u - u)$

$\Rightarrow \quad \boxed{v_2 - v_1 = 3eu} \quad (2)$

$(1) - (2) \Rightarrow \quad 3v_1 = -3eu$

$\Rightarrow \quad \boxed{v_1 = -eu}$

$\therefore \ (1) \Rightarrow \quad 0 = 2(-eu) + v_2$

$$\boxed{2eu = v_2}$$

$E_1 =$ Sum of KE before

$E_1 = \frac{1}{2}(2m)u^2 + \frac{1}{2}m(-2u)^2 = 3mu^2$

$E_2 =$ Sum of KEs after

$= \frac{1}{2} 2m(-eu)^2 + \frac{1}{2} m(2eu)^2$

$= me^2u^2 + 2me^2u^2$

$= 3me^2u^2.$

$\therefore \ \frac{E_2}{E_1} = \frac{3me^2u^2}{3mu^2}$

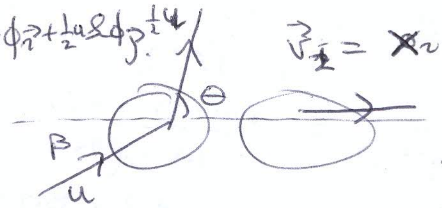
$\Rightarrow \frac{E_2}{E_1} = e^2$

$\Rightarrow \sqrt{\frac{E_2}{E_1}} = e$

1996 HLC

5(b)

$\vec{v}_1 = \frac{1}{2}u\cos\phi\,\vec{i} + \frac{1}{2}u\sin\phi\,\vec{j}$ $\qquad \vec{v}_2 = X\vec{i} + 0\vec{j}$



$e = ?$

$\vec{u}_1 = u\cos\beta\,\vec{i} + u\sin\beta\,\vec{j}$ $\qquad \vec{u}_2 = 0\vec{i} + 0\vec{j}$

$\tan\beta = \frac{1}{2}$ & $\sin\beta = \frac{1}{\sqrt5}$, $\cos\beta = \frac{2}{\sqrt5}.$

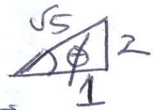
Smoothness $\Rightarrow$ $\vec{j}$ cpts unchanged

$\therefore \ u\sin\beta = \frac{1}{2}u\sin\phi$

$u\left(\frac{1}{\sqrt5}\right) = \frac{1}{2}u\sin\phi$

$\frac{2}{\sqrt5} = \sin\phi$ $\qquad \boxed{5}$

$\Rightarrow$



Pythag $\Rightarrow$

$\therefore \ \frac{1}{\sqrt5} = \cos\phi$

$\therefore \ \vec{v}_1 = \frac{u}{2\sqrt5}\vec{i} + \frac{u}{\sqrt5}\vec{j}$

$\vec{u}_1 = \frac{2u}{\sqrt5}\vec{i} + \frac{u}{\sqrt5}\vec{j}.$

PCM: $\quad m\left(\frac{2u}{\sqrt5}\right) + m(0) = m\left(\frac{u}{2\sqrt5}\right) + mX$ $\qquad \boxed{5}$

$$\boxed{\frac{2u}{\sqrt5} = \frac{u}{2\sqrt5} + X} \quad (1)$$

NLR $\quad v_2 - v_1 = -e(u_2 - u_1)$

$X - \frac{u}{2\sqrt5} = -e\left(0 - \frac{2u}{\sqrt5}\right)$

$\Rightarrow \quad \boxed{X - \frac{u}{2\sqrt5} = e\frac{2u}{\sqrt5}} \quad (2) \qquad \boxed{5}$

$(1) \Rightarrow \quad X = \frac{3u}{2\sqrt5}$ $\qquad \boxed{5}$

$(2) \Rightarrow \quad \frac{3u}{2\sqrt5} - \frac{u}{2\sqrt5} = e\frac{2u}{\sqrt5}.$

$\frac{2u}{2\sqrt5} = e\frac{2u}{\sqrt5}$

$\boxed{\frac{1}{2} = e}$ $\qquad \boxed{5}$